ES.103 – PROBLEM SHEET 0

Problem 0 .1 What is $y(x)$ [y as a function of x] given that

$$\frac{dy}{dx} = 4\frac{y}{x}$$

where you are given the boundary condition that $y = y_0$ at $x = x_0$ where y_0 and x_0 are constants.

Problem 0 .2 What is $y(x)$ given that

$$\frac{d^2y}{dx^2} + 4e^{-2x} = 0$$

where you have the boundary conditions:

$$\frac{dy}{dx} = q \quad \text{(a constant) at} \quad x = 0$$
$$y = y_0 \quad \text{(a constant) at} \quad x = 0$$

You have to integrate twice as this is a second order differential equation so you need two boundary conditions to get the complete solution. (Most people are better at differentiating then integrating so check your answer by differentiating it!)

Problem 0 .3 In physics we often only need approximate answers. The solution can usually be cast in terms of a small quantity, ϵ where ϵ is much less than 1 in value. A solution *accurate to first order in* ϵ retains terms with ϵ in but neglects terms in ϵ^2 and higher powers. For example, you might be asked to evaluate F to first order in ϵ where

$$F = \frac{(1+\epsilon)^{5/2}}{(1+2\epsilon)^3}$$

The trick here is to use the binomial expansion:

$$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \cdots$$

Your answer can always be simplified to look like

$$F \simeq 1 + C\epsilon$$

where C is a number which you should be able to compute. Try $\epsilon = 10^{-3}$ and see how it compares with the actual answer – the difference should be of order 10^{-6}, *i.e.*, ϵ^2.

Problem 0 .4 A Taylor series is something we often use and looks like

$$y(x_0 + h) = y(x_0) + h\frac{dy}{dx}(x_0) + \frac{h^2}{2!}\frac{d^2y}{dx^2}(x_0) + \cdots$$

where $\frac{dy}{dx}(x_0)$ is the first derivative of $y(x)$ evaluated at x_0. Similarly $\frac{d^2y}{dx^2}(x_0)$ is the second derivative of $y(x)$ evaluated at x_0 and so on. If h is small, we can truncate this series and get a good local approximation to $y(x)$ in the vicinity of x_0.

Try the equation $y(x) = 3x^5$ and set $x_0 = 4$ and $h = 0.1$. Evaluate y at $x = 4.1$ by using the Taylor series truncated at the second (linear) and then the third (quadratic) term and compare with the actual answer.