

ES.103 – PROBLEM SHEET 4

Problem 4.1

- What is "thermal conductivity" and what would be a typical value for earth materials
- What is a typical surface heat flow value for continents.
- What is a typical value for the radioactive volumetric heat production for continental rocks.
- What is the "reduced" heat flow

Problem 4.2 Given the following surface heat flow and heat production data for the Sierra Nevada mountains, compute the reduced heat flow q_m and the scale depth h_r .

q_s mW m ⁻²	ρH_s $\mu\text{W m}^{-3}$
18	0.3
25	0.8
25	0.9
29	1.3
31	1.5
34	2.0
42	2.6
54	3.7

You should try a *least-squares* fit to the data. This is a useful technique and can be found in many texts if you are not familiar with it.

Problem 4.3 Show that the equation governing the steady-state radial conduction of heat in a sphere or spherical shell with volumetric heat production is given by

$$\frac{k}{r^2} \frac{d}{dr} \left(r^2 \frac{dT}{dr} \right) + \rho H = 0$$

where k is the thermal conductivity, H is the heat production per unit mass and ρ is the density.

Problem 4.4 Consider one-dimensional steady-state heat conduction in a half space. The heat sources are restricted to a surface layer of thickness b , their concentration decreases linearly with depth so that $H = H_s$ at the surface $z = 0$ and $H = 0$ at $z = b$. For $z > b$, $H = 0$. There is a constant upward heat flux, q_m , from below.

What is the q_s (upward surface heat flow) - H_s relation? Determine the temperature profile as a function of z .

Problem 4.5 Assume that the radioactive elements in the earth are uniformly distributed through a near-surface layer. The surface heat flow is 70 mWm^{-2} , and there is no heat flow into the base of the layer. If $k = 4 \text{ Wm}^{-1} \text{ K}^{-1}$, $T_s = 0^\circ\text{C}$, and the temperature at the base of the layer is 500°C , determine the thickness of the layer and the volumetric heat production.