1) What is the "Peclet number" and what is its value for the mantle? Make a guess for what it might be in the core?

2) You have constructed a model of the Earth which has a core of uniform density, $\rho_c$, and a mantle of uniform density $\rho_m$. The radius, $R$, of your model is 6371 km and the core radius is $R/2$. The mean density of your model, $\bar{\rho}$, is 5500 kg m$^{-3}$ and $\rho_c = 2\bar{\rho}$.

Compute the acceleration due to gravity at the core-mantle boundary and at the surface.

Show that the pressure at the core-mantle boundary is given by

$$p\left(\frac{R}{2}\right) = \frac{26}{49} \pi G R^2 \bar{\rho}^2$$

Compute this pressure given that the gravitational constant, $G = 6.673 \times 10^{-11}$ m$^3$ kg$^{-1}$ s$^{-2}$.

3) You have almost no information about the inner core of the Earth. From seismology you have found that the seismic parameter is a function of radius given by the equation $\phi(r) = \phi_0 + B r^2$ where $\phi_0$ and $B$ are measured constants. ($\phi = V_p^2 - 4/3V_s^2$). You also know that density must decrease slightly with radius in the inner core (because the pressure is decreasing) so you approximate $\rho(r)$ by $\rho(r) = \rho_0 + C r^2$ where $\rho_0$ and $C$ are unknown constants.

Assuming that the inner core is adiabatic and homogeneous, compute $\rho_0$ and $C$ given that $\phi_0 = 110$ km$^2$ s$^{-2}$, $B = -3 \times 10^{-6}$ and the gravitational constant, $G = 6.673 \times 10^{-11}$ m$^3$ kg$^{-1}$ s$^{-2}$.

If $\phi(r)$ is only determined with a precision of $\sim 1\%$, estimate by how much your density profile can vary and so determine if this is a good way to estimate $\rho(r)$ in the inner core. Furthermore, you have essentially assumed that the inner core is convecting. Do you think this is likely?

[Hint – you may assume that $C r^2 / \rho_0 \ll 1$]

4) Estimate the adiabatic temperature rise across the lower mantle and across the outer core.